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A new car-following model: full velocity and acceleration difference model

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Abstract. By introducing acceleration difference terms into the full velocity difference models (FVDM) by Jiang et al. (1995), we present a full velocity and acceleration difference model (FVADM). The main improvement upon the previous models is that the FVADM can exactly describe the driver's behavior under an urgent case, where no collision occurs and no unrealistic deceleration appears in this model, while vehicles determined by the previous car-following models collide after only few seconds. The model is investigated by numerical methods. The simulation results indicate that the acceleration difference has an important impact on the traffic dynamics, especially under urgent conditions. Besides the urgent situations, the model still remains similar properties to those of the FVDM. In the model, the phase transition of traffic flow is observed, and the hysteresis loop is obtained in the headway- velocity plane, also.

PACS. 89.40.-a Transportation – 64.60.Cn Order-disorder transformations; statistical mechanics of modelsystems – 02.60.Cb Numerical simulation; solution of equations – 05.70.Fh Phase transitions: general studies

1 Introduction

In recent years, car-following models, which describe the processes in which drivers follow each other in the traffic stream, now form the cornerstone for many important research areas, including (a) simulation modelling, where the car-following models control the motion of the vehicles in network, and (b) the functional definition of advanced vehicle control and safety systems (AVCSS), which are being introduced as a driver safety aid in an effort to mimic driver behavior but remove potential hazards that may occur [1].

Car-following model is based on the idea that each driver controls a car under the stimuli from the preceding car, which can be expressed by the function of headway distance or the relative velocity of two successive cars. Conventional car-following models have a difficulty in describing both free flow and the congested flow separated by the jamming transition on unified ground [2]. The car-following models have been extensively developed by attempting to increase the realism of the driver behaviors [3–15].

Bando et al. proposed a simple car-following model, the optimal velocity model (OVM), which described many properties of real traffic flows and was easily interpretable [11]. In this model, each vehicle is described by a simple differential equation using the OV function, which is dependent on the headway distance, and each driver controls the velocity based on the OV function. It is found that the OV model can successfully describe the jamming transition. Helbing and Tilch made a calibration of the OVM with respect to the empirical data [13]. The comparison with field data suggests that high acceleration and unrealistic deceleration occur in the OVM.

In order to make an improvement in the OVM, Helbing and Tilch developed a generalized force model (GFM) [13]. A velocity difference term is taken into account in the model when the velocity of the following vehicle is larger than that of the leader (that is negative velocity difference). The simulation results show that the GFM is in good agreement with the empirical data.

However, neither the OVM nor the GFM can explain the traffic phenomena described by Treiber et al. [15]. If the preceding cars are much faster, then the vehicle would not brake, even if its headway is smaller than the safe distance. Jiang et al. pointed out that the GFM exhibited poor delay time of car motion and kinematic wave speed at jam density because of neglecting the effects of positive velocity differences on the traffic dynamics [14]. By taking both positive and negative velocity differences into account, Jiang et al. proposed a full velocity difference model (FVDM). The numerical investigations indicated that the FVDM could describe the phase transition of traffic flow and estimate the evolution of traffic congestion.

Both the GFM and the FVDM can avoid an accident if a freely moving car from a large distance reaches a

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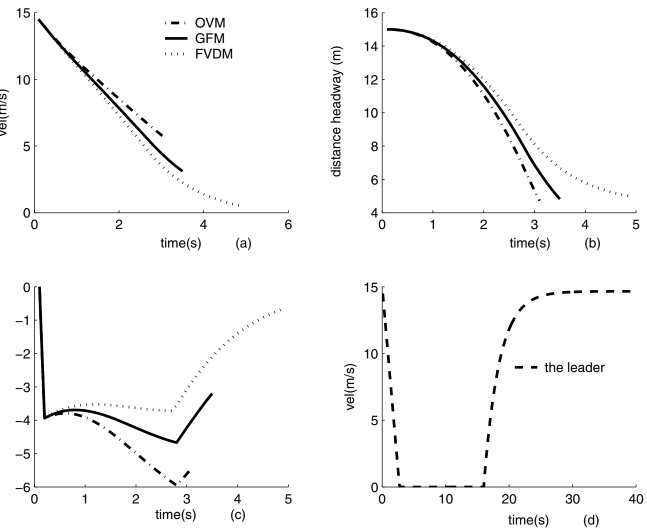


Fig. 1. The simulations in the OVM, GFM, and FVDM under an urgent case mentioned in the text (a) velocity of the follower, (b) headway distance of the follower, (c) acceleration of the follower, (d) the velocity of the leader.

standing car. In this situation, the velocity difference has strong effects on the traffic behavior due to its large value, so that the moving vehicle can quickly respond to the standing vehicle ahead. However, if the successive vehicles have nearly identical speed, there is zero or small velocity difference, whether the follower can react correctly to the strong decelerating leader to avoid a collision. Let us investigate an urgent case according to the above models. The urgent case can be defined as: a situation that the preceding car decelerates strongly, if two successive cars move forward with much small headway-distance, e.g. a freely moving car decelerates drastically for an accident in front or the red traffic light at an intersection, the following car is freely moving and the distance between the two cars is quite small. The simulation under such situation is carried out in the following. The model parameters are the same as those in reference [14]. There are two cars free moving with identical speed 14.5 m/s at initial time $t = 0$ in a road. The initial headway distance is 15 m. The leader decelerates with the rate –5.5 m^2/s until it stops completely. The leader remains standing for several seconds before accelerating back to its original speed.

The simulation results in the OVM, GFM and FVDM are shown in Figures 1a, 1b, 1c, which demonstrate the variation of the velocity, headway distance and acceleration of the follower, respectively. Figure 1d shows the velocity of the leader. It can be observed that the leader and the follower collides at time $t = 3.1$ s, 3.5 s, and 4.9 s in the OVM, GFM and FVDM, respectively. It indicates that the velocity difference is not enough to avoid an accident under such urgent case. We argue that the effect of the acceleration difference is not considered in these models. The follow-the leader data, including the vehicle speed, the netto distance to the car in front, the relative velocity and the acceleration have been recorded by a research group of the Bosch GmbH [16]. By a correction analysis, they pointed out that among all possible combinations of subsets of the four quantities, the former three were the most significant variables for description of vehicle dynamics. However, in their field data, the acceleration difference between the leader and the follower were neglected. In fact, if the processor brakes instantaneously, its lighting brake lights can be observed by the following driver, otherwise the lights are dark. The following driver often considers the acceleration difference with its preceding car in deciding his driving state at the next time step, especially in the urgent braking cases. Therefore, we argue that the acceleration difference plays an important role in traffic dynamics. Other microscopic models have been extended to describe these realistic traffic behaviors [17–19]. To our knowledge, there are seldom studies on car-following models.

By taking the acceleration difference into account, we develop a new car-following model, based on the OVM and the FVDM. Since our model incorporates both the velocity difference and acceleration difference, we call it full velocity and acceleration difference model (FVADM). The FVDAM can manage the urgent case described above and car collisions do not happen any more. Simultaneously, the traffic phenomena described by Treiber et al. [15] can be explained by the FVADM. Our simulation results also show that the acceleration difference has important effects on traffic dynamics, and similar to the FVDM the model can describe the phase transition of traffic flow and estimate the evolution of traffic congestion. The paper is organized as follows: the previous car-following models, including the OVM, the GFM and the FVDM are reviewed in Section 2. In Section 3 the FVADM is presented and the comparisons among the previous three models and the FVADM are made. The property of our model using numerical methods is investigated in Section 4. Finally, conclusions are summarized.

2 Previous car-following models

2.1 The optimal velocity

In 1995, Bando et al. presented a simple car-following model called the optimal velocity model (OVM) [11]. The central idea of the OVM is the introduction of the optimal velocity (OV) function, which determines the optimal velocity (safety velocity) according to the distance-headway. The motion of a vehicle i is described by the following equation [11],

$$\frac{dv_i(t)}{dt} = \kappa \left[V\left(s_i(t)\right) - v_i(t)\right]$$
$$s_i(t) = x_{i+1}(t) - x_i(t) - l_{i+1} \tag{1}$$

where $v_i(t) > 0$ and $x_i(t) > 0$ are the velocity and position of the ith vehicle at time t, $s_i(t)$ denotes the netto distance between the ith vehicle and its front vehicle $i+1$ at time t, κ is the sensitivity of the driver, V is the OV function and l_{i+1} denotes the length of vehicle $i+1$, which is usually chosen as 5 m in the simulations. The OVM has been calibrated with the empirical data by Helbing and Tilch. The OV function is adopted as [13],

$$V(s) = V_1 + V_2 \tanh(C_1 s - C_2). \tag{2}$$

The resulting optimal parameter values are $\kappa = 0.85\,\mathrm{s}^{-1}$, $V_1 = 6.75$ m/s, $V_2 = 7.91$ m/s, $C_1 = 0.13$ m^{-1}, and $C_2 = 1.57$. The comparison with field data suggests that high acceleration and unrealistic deceleration appear in the OVM.

2.2 The generalized force model

Helbing and Tilch [13] proposed a generalized force model (GFM) for overcoming the shortage of the OVM. Its formulation is as follows,

$$\begin{aligned} \frac{dv_i(t)}{dt} &= \kappa\left[v_m - v_i(t)\right] + \kappa\left[V(s_i(t)) - v_m\right] \\ &\quad + \lambda\Theta(-\Delta v_i(t))\,\Delta v_i(t) \\ \Delta v_i(t) &= v_{i+1}(t) - v_i(t) \end{aligned} \tag{3}$$

where v_m is the maximum speed, Θ is the Heaviside function, $\Delta v_i(t)$ is the velocity difference between the preceding vehicle $i+1$ and the following vehicle i. In the GFM, the calibration shows $\kappa = 0.41\ \mathrm{s}^{-1}$, which is much smaller than that in the OVM.

2.3 The full velocity difference model

By taking the positive velocity difference into account, Jiang et al. [14] developed a full velocity difference model (FVDM), based on the OVM and the GFM. The dynamic equation of a vehicle i is as follows,

$$\frac{dv_i(t)}{dt} = \kappa\left[V(s_i(t)) - v_i(t)\right] + \lambda\Delta v_i(t)\,. \tag{4}$$

Here, λ is chosen as a step function,

$$\lambda = \begin{cases} a, & s \le s_c \\ b, & s > s_c \end{cases} \tag{5}$$

where parameters a, b, and s_c are taken as $a = 0.5\ \mathrm{s}^{-1}$, $b = 0$ and $s_c = 100$ m. The FVDM considers more aspects in car-following process than the OVM and the GFM. However, it still has some problems in depicting driver dynamic behaviors under some urgent conditions.

3 The full velocity and acceleration velocity model

In the real traffic, if the vehicle decelerates dramatically, the brake lights are alight, otherwise they are dark. Its following driver can be aware of the strong deceleration by observing the lighting brake lights of the preceding vehicle, and react rapidly to avoid collision. That's the reason why so many frequent urgent brakes don't cause mass of traffic accidents in our life. According to our analysis in Section 1, we extend the FVDM by incorporating the acceleration difference, and then get a new model, the full velocity and acceleration difference model (FVADM). The dynamic equation is described as follow:

$$\begin{aligned} \frac{dv_i(t)}{dt} &= \kappa\left[V(s_i(t)) - v_i(t)\right] + \lambda\Delta v_i(t) \\ &\quad + kg\left(\Delta a_i(t-1), a_{i+1}(t)\right)\Delta a_i(t-1) \\ \Delta a_i(t) &= a_{i+1}(t) - a_i(t) = \frac{dv_{i+1}(t)}{dt} - \frac{dv_i(t)}{dt} \\ g\left(\Delta a_i(t-1), a_{i+1}(t)\right) &= \begin{cases} -1, \Delta a_i(t-1) > 0 \\ \quad \text{and}\, a_{i+1}(t) \le 0 \\ 1,\ \text{others} \end{cases} \end{aligned} \tag{6}$$

where $\Delta a_i(t)$ is the acceleration difference between the preceding vehicle $i+1$ and the following vehicle i. Parameter λ is the same as equation (5). Similar to λ, we take the following step function for k,

$$k = \begin{cases} c, & s \le s_c \\ d, & s > s_c \end{cases} \tag{7}$$

where parameters c, d are taken as $c = 0.5$, $d = 0$. Function $g(\cdot)$ is to determine the sign of the acceleration difference term. The acceleration difference term brings a decelerating impact on the follower, if the acceleration difference is positive and the preceding car decelerates. Let us discuss the behavior of the following vehicle in the FVADM under this situation as follows: (a) the follower will decelerate rapidly due to adding the acceleration difference into the model, if both the velocity difference and the netto distance between the leader and the follower are exceedingly small, which can be viewed as an urgent case. Therefore, the accident would not happen under urgent cases in the FVADM. However, in urgent cases there is no enough deceleration to prevent collisions in previous model. (b) The follower would not decelerate if the preceding vehicle has much larger speed, even if the netto distance is less than the safe distance. That means that the FVADM can explain the traffic phenomena described by Treiber et al. [15].

Equation (6) can be rewritten as the following form,

$$\begin{aligned} \frac{dv_i(t)}{dt} &= \kappa\left[v_m - v_i(t)\right] + \kappa\left[V(s_i(t)) - v_m\right] \\ &\quad + \lambda\Theta(-\Delta v_i(t))\,\Delta v_i(t) + \lambda\Theta(\Delta v_i(t))\,\Delta v_i(t) \\ &\quad + k\,g(\Delta v_i(t))\,\Delta a_i(t-1)\,. \end{aligned} \tag{8}$$

From the equation (8), it can be found that our model is reduced to the FVDM if $k = 0$, and reduced to the GFM if positive Δv_i is neglected and $k = 0$, reduced to the OVM if $\lambda = 0, k = 0$.

4 Simulations

Firstly, we apply the FVADM to simulate the vehicle behaviors under the same urgent case described in Section 1.

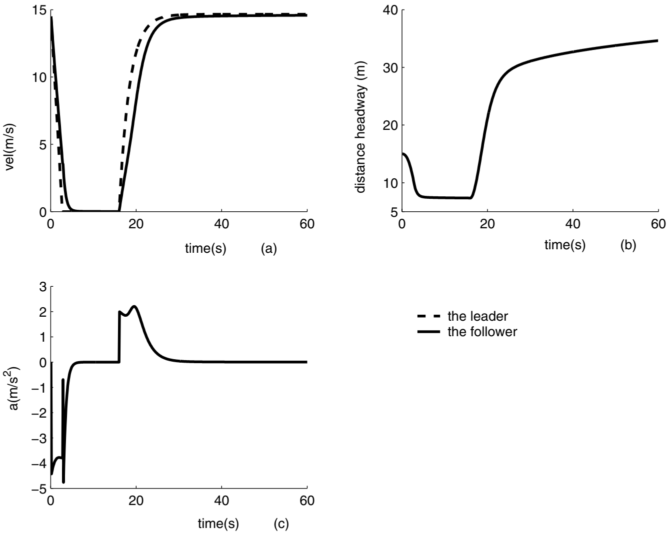


Fig. 2. The simulations in the FVADM under the urgent case same as that in Figure 1. (a) Velocity of the follower and the leader, (b) headway distance of the follower, (c) acceleration of the follower.

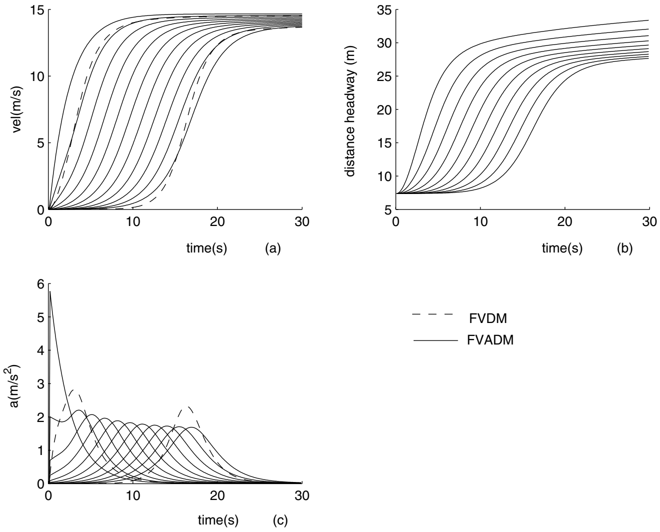


Fig. 3. The starting motion of vehicles ($N = 11$) in the FVADM ($\lambda = 0.5$, and $k = 0.5$); (a) velocity, (b) headway distance, (c) acceleration.

The results are shown in Figure 2. It can be seen that no accident occurs and no unrealistic deceleration appears in our model. There is a maximal deceleration –4.76 m^2/s. According to our driving experience in the life, it is possible in some urgent situations. Due to the effect of acceleration difference, the follower can react rapidly to the leader's sudden behavior. From this point of view, the FVADM describes the car-following dynamics more exactly than the previous models.

Secondly, the delay time δt of car motion and the kinematic wave speed c_j at jam density are examined in the FVADM. We carry out the same simulation as that in reference [14]. First a traffic signal is yellow and all cars are waiting with headway 7.4 m, where the OV function is zero. Then at time $t = 0$, the signal changes to green and cars start. Figures 3a, 3b show the variation of the velocity and headway distance of all vehicles in our model (those of the OV, the GFM, the FVDM can be found in reference [14] (Figs. 1a, 1b, 1c). The variations of all vehicles' acceleration in the simulation above are shown in Figure 3c. The solid line shows the diagram of our model, and the dashed line represents that of the first and the last follower in the FVDM. From Figures 3a, 3c, the followers of our model at the initial stage have faster start than those of the FVDM, while the status reverses after the stage. This completely fits the real driving behavior, the following drivers initially have strong desires to start moving forward, and they gradually calm down with the development of time. The values of δt and c_j are 1.41 s and 18.86 km/h. It is found that the FVADM predicts correct delay of car motion and kinematic wave speed in jam density, since Bando et al. [12] claimed that the observed δt is of the order of 1 s, and Del Castillo and Benitez [20] indicated that c_j ranged between 17 and 23 km/h. Then let us study whether the model causes unrealistically high acceleration just as the OVM. From Figure 3c, it can be seen that the maximum value of acceleration in the FVADM isn't beyond the limitation of 4 m^2/s.

Thirdly, a numerical simulation is carried out to observe the traffic dynamics in the FVADM, similar to reference [14]. There are $N = 100$ cars running on a road with the length $L = 1500$ m, under a periodic boundary condition. We set the same initial disturbance as that in reference [14]

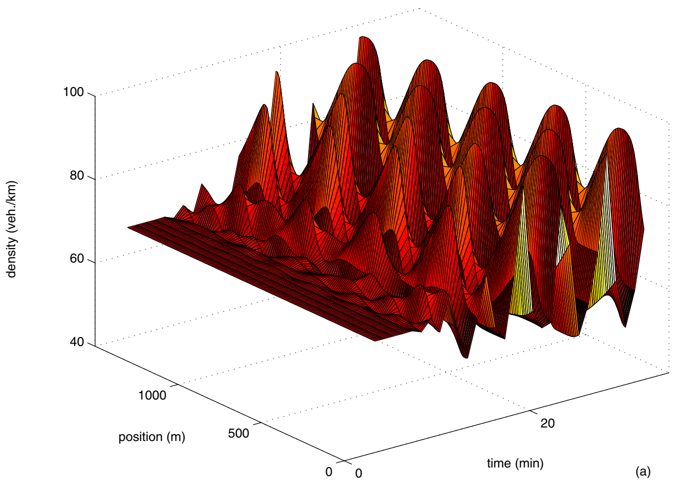
$$x_1(0) = 1\,\mathrm{m};\; x_i(0) = (i-1)\,L/N\,\mathrm{m},\ \mathrm{for}\ i \neq 1 \tag{9}$$

$$v_i(0) = V(L/N). \tag{10}$$

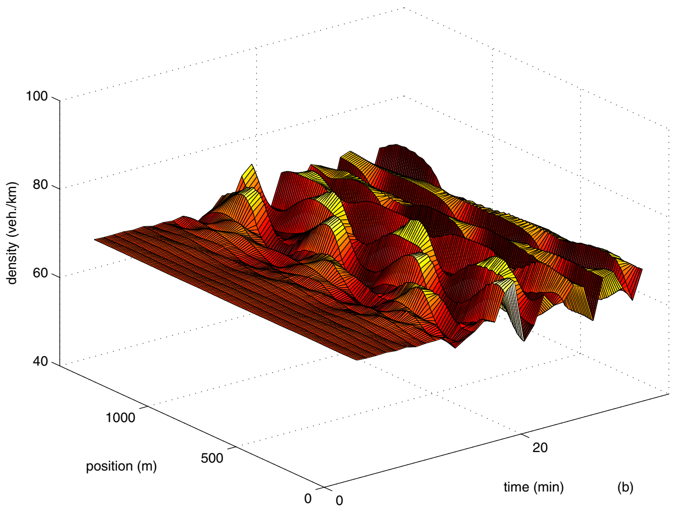
We set $\lambda = 0.5$, and $k = 0.5$, and other parameters are the same as the previous simulation. The simulation is carried out for 2000 s with the evolution of the time.

Figures 4a, 4b show the spatio-temporal density of traffic states, according to the FVDM and the FVADM respectively. The phase transit from free flow to congested traffic is observed in the two models. However, congested traffic flows in the FVDM are denser and wider than those of the FVADM, indicating that more serious congestion occurs in the FVDM. This effect attributes to the acceleration difference. The results suggest that rapid reaction of drivers in the FVADM eliminates the denser and larger jam, appearing in the FVDM. This coincides with the real traffic.

In order to get further insight information of the variation of vehicles, the velocity configurations of all vehicles at $t = 500$ s and $t = 2000$ s are shown in Figures 5a, 5b. It is observed that the traffic flow transforms from free flow at an initial stage to congested traffic with the development of the time, in both models. The velocity at congestion is close to zeros in the FVDM, where the vehicles almost stop, while it is near 1.4 m/s in the FVADM, where the vehicles move forward slowly. Since the occurrence of the traffic jam can be recognized in the headway-velocity plane, the value of the headway and velocity of a



(a)



(b)

Fig. 4. Spatiotemporal density plots of traffic states; (a) the FVDM ($\lambda = 0.5$), (b) the FVADM ($\lambda = 0.5$, and $k = 0.5$).

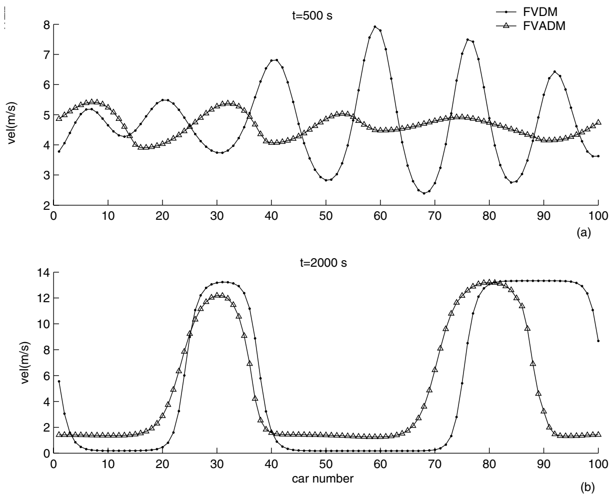


Fig. 5. Velocity of all vehicles for the FVDM ($\lambda = 0.5$) and the FVADM ($\lambda = 0.5$, and $k = 0.5$); (a) $t = 500$ s, (b) $t = 1000$ s.

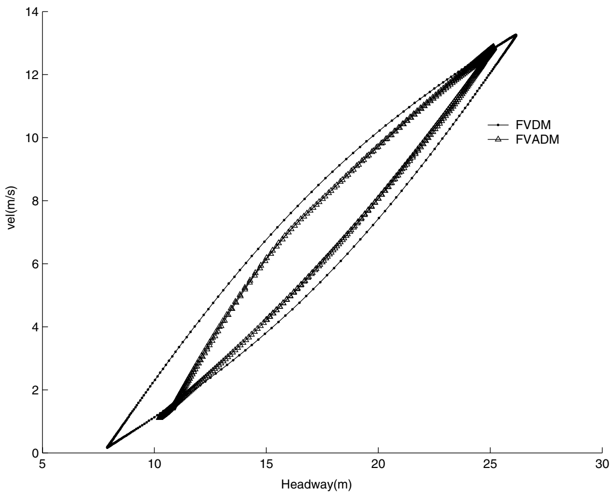


Fig. 6. Headway-velocity plots for the FVDM and the FVADM.

tested car after 1000 s is depicted in Figure 6. The velocity-headway trajectory in the FVADM is compared to that in the FVDM. A closed trajectory called a hysteresis loop is clearly observed in both models, which is a characteristic of the jam. When a vehicle decelerates, its velocity in the FVADM is higher than those determined by the FVDM for the same headways, which is an effect we ascribe to the acceleration difference in the FVADM. Accelerating vehicle in the FVADM has larger headway at a given velocity due to the effect of the acceleration difference. The simulation results directly indicate that the acceleration difference term has an important impact on the traffic dynamics.

5 Conclusions

By introducing the acceleration difference into the FVDM, a full velocity and acceleration difference model (FVADM) is proposed. The key improvement upon the previous models is that the FVADM can exactly describe the driver's behavior under an urgent case, where no unrealistic deceleration appears and no collision occurs. However, in the same situation, vehicles crash soon in the previous models. Our model is investigated by numerical methods. The simulation results reveal that the acceleration difference lessens the denser and wider jam and plays an important role in the traffic dynamics. In addition, similar to the FVDM, the reasonable delay time of car motion and kinematic wave speed at jam density are obtained in the FVADM. The phase transition from free flow to the congestion is observed, and the hysteresis loop is obtained in this model, also.

All original code in this work can be provided if being needed.

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